1 Job Search, Wage Determination and Equilibrium Unemployment

The labour markets have frictions and the match between workers looking for a job and firms looking to hire a worker is not instantaneous nor certain. In the economy there are present at the same time unemployed workers U and vacancies V: job matches M are created according to a matching function. In this circumstance we have seen that there exist a relationship between the vacancies rates v and the unemployment rate u for which total unemployment is constant. This relationship is known is known as the Beveridge Curve and, under constant return to scale of the matching function, it can be derived as:

$$v = \left(\frac{\gamma}{K}\right)^{\frac{1}{1-\beta}} \frac{(1-u)^{\frac{1}{1-\beta}}}{u^{\frac{\beta}{1-\beta}}}$$

1.1 Workers Utility

Suppose we have identical individuals living forever.

Workers' lifetime utility is given by the sum of their discounted incomes x (r is the discount rate):

$V = \sum_{t=0}^{+\infty} \frac{x_t}{(1+r)^t}$.

The value of the above is equivalent to a perpetual rent, and therefore we have

$V = \frac{x_t}{r}$

Each individual can be in either of 2 states, he can be employed earning w per period, or he can be unemployed earning b (with $w > b$): the payments are postponed at the end of the period. Strong assumption: w is fixed. The parameter b is what the worker get while unemployed: this may be an unemployment benefits but could also entails other components like housework or earning from informal jobs.

In each period an employed worker may lose (or give up) his job with probability γ, an unemployed worker receive a job offer with probability λ. To all extents γ is the probability of leaving a job while λ is the probability of receiving a job offer.

The value of the being employed is then given by:

$V_E = w\Delta + (1-\gamma\Delta)\frac{V_E}{1+r\Delta} + \gamma\Delta\frac{V_U}{1+r\Delta}$

$V_U = b\Delta + \lambda\Delta\frac{V_E}{1+r\Delta} + (1-\lambda\Delta)\frac{V_U}{1+r\Delta}$

$\frac{V_E}{\Delta}(1+r\Delta) = w(1+r\Delta) + (1-\gamma\Delta)\frac{V_E}{\Delta} + \gamma V_U$

$\frac{V_E}{\Delta} + rV_E = w(1+r\Delta) + \frac{V_E}{\Delta} - \gamma V_E + \gamma V_U$

for $\Delta \to 0$

$$rV_E = w + \gamma(V_U - V_E)$$

and, through a similar procedure,

$$rV_U = b + \lambda (V_E - V_U)$$

We can then obtain the increase of utility $(V_E - V_U)$ due to a change of status from unemployment to employment

$V_E - V_U = \frac{w + \gamma (V_U - V_E) - b - \lambda (V_E - V_U)}{r} = \frac{w - b - (\lambda + \gamma)(V_E - V_U)}{r}$

$r (V_E - V_U) + (\lambda + \gamma)(V_E - V_U) = w - b$

$$(V_E - V_U) = \frac{w - b}{(r + \lambda + \gamma)}$$

We can also use the above to compute the lifetime utility (asset value) of currently being employed.

$rV_E = w - \gamma \frac{w - b}{(r + \lambda + \gamma)}$

$$V_E = \frac{(r + \lambda) w + \gamma b}{r (r + \lambda + \gamma)}$$

1.2 Firm Side

We can obtain the flow value of utility with a similar procedure:

$$rV_F = A - w - C + \gamma (V_V - V_F)$$

$$rV_V = -C + \alpha (V_F - V_V)$$

where A is the production in a filled firms and C is an operating costs that has to be paid in both states. The parameter α is the (instant) probability of filling the vacancy.

We can then obtain the increase of utility $(V_F - V_V)$ due to a change of status from vacant to filled (same procedure as for workers).

$$V_F - V_V = \frac{A - w}{r + \alpha + \gamma}$$

1.3 Wage Determination

Whenever a firm and a worker form a match they earn from the increase in their utility, then they are both better of. We assume they choose to split the gain (other assumptions are also possible), and this determines the wage:

$V_F - V_V = V_E - V_U$

$\frac{w - b}{r + \lambda + \gamma} = \frac{A - w}{r + \alpha + \gamma}$

$w (r + \alpha + \gamma) - b (r + \alpha + \gamma) = A (r + \lambda + \gamma) - w (r + \lambda + \gamma)$

$w (r + \alpha + \gamma + r + \lambda + \gamma) = A (r + \lambda + \gamma) + b (r + \alpha + \gamma)$

$$w = \frac{r+\gamma+\lambda}{2r+2\gamma+\alpha+\lambda}A + \frac{r+\alpha+\gamma}{2r+2\gamma+\alpha+\lambda}b$$

The actual value of wage depends on productivity A but also on the parameters describing the search process $(\gamma, \alpha, \lambda)$ and on the outside option of workers b.

1.4 Equilibrium condition

We assume that product market is perfectly competitive with free entry. Thus, the free entry condition implies that the value of a vacant firm must be zero.

FREE ENTRY CONDITION $rV_v = 0$

$rV_V = -C + \alpha\left(V_F - V_V\right) = 0$

$rV_V = -C + \alpha\left(\dfrac{A-w}{r+\alpha+\gamma}\right) = 0$

We can now insert wage in the above equation.

$rV_V = -C + \dfrac{\alpha}{r+\alpha+\gamma}\left(A - \dfrac{r+\gamma+\lambda}{2r+2\gamma+\alpha+\lambda}A - \dfrac{r+\gamma+\alpha}{2r+2\gamma+\alpha+\lambda}b\right) = 0$

$rV_V = -C + \dfrac{\alpha}{r+\alpha+\gamma}\left(\dfrac{r+\gamma+\alpha}{2r+2\gamma+\alpha+\lambda}A - \dfrac{r+\gamma+\alpha}{2r+2\gamma+\alpha+\lambda}b\right) = 0$

$rV_V = -C + \left(\dfrac{\alpha}{2r+2\gamma+\alpha+\lambda}\right)(A-b) = 0$

$$\frac{2r+2\gamma+\lambda}{\alpha} = \frac{A-b-C}{C}$$

when the above condition is not met the value of a vacant firm is different from zero and thus firms either enter or leave the markets. Note that the parameters λ and α should still depends on the matching mechanisms!

1.5 The role of the Matching Function

In our setup actual match depends on the number U of unemployed workers looking for a job and the number V of vacant firms looking for a worker to hire. The number of matches is determined by a function $M()$ representing the matching technology.

$$M = M(U,V) = KU^{\beta}V^{1-\beta}$$

with $\beta < 1$. The parameter K measures the technology of matching.

According to the above matching funtion we have that the probability of finding a job λ is given by $\lambda = \dfrac{M(U,V)}{U}$ and the probability to fill a vacancy is $\alpha = \dfrac{M(U,V)}{V}$. Therefore we have that

$\lambda = \dfrac{M(U,V)}{U} = \dfrac{KU^{\beta}V^{1-\beta}}{U} = K\left(\frac{V}{U}\right)^{1-\beta}$

$$\lambda = K(\theta)^{1-\beta}$$

$$\alpha = \frac{M(U,V)}{V} = \frac{KU^{\beta}V^{1-\beta}}{V} = K\left(\tfrac{V}{U}\right)^{-\beta}$$

$$\alpha = K(\theta)^{-\beta}$$

The above equations for λ and α can be inserted in the no-entry condition. We obtain

$$\frac{A-b-C}{C} = \frac{2r+2\gamma}{\alpha} + \frac{\lambda}{\alpha}$$

$$\frac{A-b-C}{C} = \frac{2r+2\gamma}{K(\theta)^{-\beta}} + \frac{K(\theta)^{1-\beta}}{K(\theta)^{-\beta}}$$

$$\frac{A-b-C}{C} = \frac{2r+2\gamma}{K}\theta^{\beta} + \theta$$

1.6 Equilibrium Employment

In equilibrium, the free entry condition implies that

$$\frac{A-b-C}{C} = \frac{2r+2\gamma}{K}\theta^{\beta} + \theta.$$

The above equation could be used to obtain a precise value of θ and inserting such value in the equation of the Beveridge Curve we would obtain the equilibrium value of u and v. In any case even if we do not solve the above equation we can still analyse the effect of changes of the exogenous components on the unemployment equilibrium level.

if $K \uparrow$ the left hand side increases: then θ must increase to mantain the free entry condition. Combining this increase of θ with the Beveridge Curve (see Figure 1) we see that equilibrium employment decreases

if $\gamma \uparrow$ the right hand side increases: then θ must decrease to mantain the free entry condition. Combining this decrease of θ with the Beveridge Curve (see Figure 1) we see that equilibrium employment increases.

if $b \uparrow$ the left hand side decreases: then θ must decrease to mantain the free entry condition. Combining this decrease of θ with the Beveridge Curve (see Figure 1) we see that equilibrium employment increases.

Curve and equilibrium employment.png

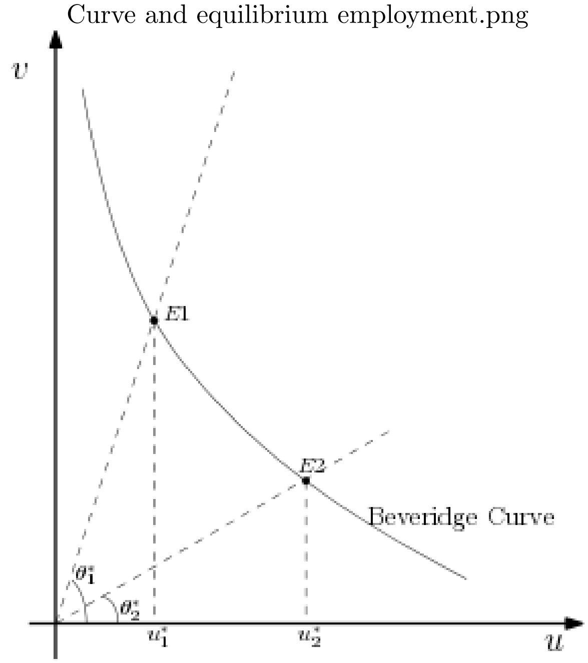

Figure 1: Changes in θ and changes in equilibrium unemployment