1 Job Search, Unemployment Duration and Unemployment Benefits

Wages are drawn from a distribution with cumulative density function $F(w)$ (and probability density function $f(w)$). Simplifying assumption (only for computation): jobs last forever ($\gamma = 0$).

1.1 Flow utility of unemployment

Let's define V and U as the asset value of the two states (employment and unemployment).

Under the above assumptions we have.

$$rV(w) = w$$

$$rU = b + \lambda \int_0^{+\infty} \max\{0, V(w) - U\} \, dF(w)$$

1.2 Reservation Wage

Reservation wage w_r is defined as the wage for which employment and unemployment bestow the same utility. As such, no worker will accept a wage that is lower than the reservation wage.

$$w_r : V(w_r) = U$$

and given that $rV(w) = w$ we have $rV(w_r) = w_r$ and $w_r = rU$

$w_r = b + \lambda \int_0^{+\infty} \max\{0, V(w) - U\} \, dF(w)$

$w_r = b + \lambda \int_0^{w_r} \max\{0, V(w) - U\} \, dF(w) + \lambda \int_{w_r}^{+\infty} \max\{0, V(w) - U\} \, dF(w)$

$w_r = b + \lambda \int_{w_r}^{+\infty} [V(w) - U] \, dF(w)$

$w_r = b + \frac{\lambda}{r} \int_{w_r}^{+\infty} [rV(w) - rU] \, dF(w)$

$$w_r = b + \frac{\lambda}{r} \int_{w_r}^{+\infty} (w - w_r) \, dF(w)$$

We want to show that $w_r = b + \frac{\lambda}{r} \int_{w_r}^{+\infty} (w - w_r) \, dF(w) = b + \frac{\lambda}{r} \int_{w_r}^{+\infty} [1 - F(w)] \, dw$.

To do that we have to go through several steps (**DIFFICULT DIMONSTRATION**).

First consider that, integrating by substitution:

$w_r = b + \frac{\lambda}{r} \int_{w_r}^{+\infty} (w - w_r) \, dF(w)$

$w_r = b + \frac{\lambda}{r}\int_{w_r}^{+\infty} (w - w_r)\, \frac{dF(w)}{dw} dw$

then we integrate by part (the general rule is $\int_c^d a(x)b'(x)dx = [a(x)b(x)]_c^d - \int_c^d a'(x)b(x)dx$), considering a general upper bound $\overline{w}$:

$\int_{w_r}^{\overline{w}} (w - w_r)\, \frac{dF(w)}{dw} dw$

$[(w - w_r)\, F(w)]_{w_r}^{\overline{w}} - \int_{w_r}^{\overline{w}} F(w)\, dw$

$(\overline{w} - w_r)\, F(\overline{w}) - \int_{w_r}^{\overline{w}} F(w)\, dw$

$\int_{w_r}^{\overline{w}} F(\overline{w})\, dw - \int_{w_r}^{\overline{w}} F(w)\, dw$

$\int_{w_r}^{\overline{w}} [F(\overline{w}) - F(w)]\, dw$

for $\overline{w} \to \infty$ we have (consider that $F(\infty) = 1$):

$$\int_{w_r}^{\infty} [1 - F(w)]\, dw$$

and thus

$$w_r = b + \frac{\lambda}{r}\int_{w_r}^{+\infty} [1 - F(w)]\, dw$$

For $\gamma \neq 0$ we simply have

$$w_r = b + \frac{\lambda}{r + \gamma}\int_{w_r}^{+\infty} [1 - F(w)]\, dw$$

It follows that an effect of b on the reservation wage is positive.

1.3 Unemployment Duration

The instant probability H of accepting a job offer is given by the probability of receiving a job offer (λ) multiplied by the probability of accepting that offer $(1 - F(w_r))$:

$H = \lambda [1 - F(w_r)]$ is the instant probability

Given H we have that

e^{-Ht} is the probability of still being unemployed at the instant t

and

He^{-Ht} is the probability of leaving employment exactly at time t

Duration is then given by

$D = \int_0^{+\infty} tHe^{-Ht}dt$

We want to show that

$D = \int_0^{+\infty} tHe^{-Ht}dt = \frac{1}{H}$

to see that, we integrate by parts (the general rule is $\int_c^d a(x)b'(x)dx = [a(x)b(x)]_c^d - \int_c^d a'(x)b(x)dx$):

(**DIFFICULT DIMONSTRATION**).

$H\int_0^{+\infty} te^{-Ht}dt = H\left\{\left[t\frac{e^{-Ht}}{-H}\right]_0^{+\infty} - \int_0^{+\infty} \frac{e^{-Ht}}{-H}dt\right\} = H\left\{\left[t\frac{e^{-Ht}}{-H}\right]_0^{+\infty} - \left[\frac{e^{-Ht}}{H^2}\right]_0^{+\infty}\right\} = H\left\{0 - \left[-\frac{1}{H^2}\right]\right\} = \frac{1}{H}$

note that $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x}$ (applying the Hopital rule).

The effect of b on duration: it increases reservation wages and through them unemployment duration

1.4 Search Effort

Consider that actual instant utility is $b - g(s)$ where $g(s)$ is a positive function of the search effort s.

$g'(s) > 0$

$g''(s) > 0$

The probability of receiving a job offer is now $s\lambda$ and then:

$w_r = b - g(s) + \frac{s\lambda}{r+\gamma}\int_{w_r}^{+\infty} [1 - F(w)]\, dw$

Given that $rU = w_r$, an unemployed individual maximises his utility maximising w_r. The optimal search effort is then given by

$g'(s) = \frac{\lambda}{r+\gamma}\int_{w_r}^{+\infty} [1 - F(w)]\, dw$

It follows that an increase in b still increases the reservation wages inducing a reduction of the right hand side. The left hand side have to increase and thus s decreases:

unemployment benefits reduce search effort and increase unemployment duration.