Employment protection with fixed wages

Firms operate in two period. In the first one production *Y* depends on employment *N* according to the following function $Y = \log N$.

In the second period instead production is $Y = A \log N$ where A is a productivity parameter which is stochastic: in particular with probability *p* we have is $A_L < 1$ and with *1-p* it is $A_H > 1$. Wage is fixed at *W* in both periods and price are normalized (and fixed) to 1. Firing costs are equal to *z*

We then have the following expected profits $E(\pi)$ over the two periods:

$$E(\pi) = Y_1 - WN_1 + E[Y_2 - WN_2 - z \cdot \max(N_1 - N_2, 0)] \tag{1}$$

And specifying the production functions:

$$E(\pi) = \log N_1 - WN_1 + pA_L \log N_{2,L} + (1-p)A_H \log N_{2,H} - p \cdot WN_{2,L} - p \cdot z \cdot \max\left(N_1 - N_{2,L}, 0\right) - (1-p) \cdot WN_{2,H} - (1-p) \cdot z \cdot \max\left(N_1 - N_{2,H}, 0\right) \tag{2}$$

Where $N_{2,L}$ and $N_{2,H}$ are the employment that is (optimally) chosen in case of low or high productivity.

We solve it with backward induction.

Second period decisions:

If we have A_H (with probability *1-p*), the firms necessarily want to increase N_1 and $N_1 - N_2 < 0$. Then second period profits are:

$$\pi_2 = A_H \log N_2 - WN_2 - z \cdot \max(N_1 - N_2, 0) = A_H \log N_2 - WN_2 \tag{3}$$

In this case optimal employment entails

$$\frac{\partial \pi_2}{\partial N_2} = \frac{A_H}{N_2} - W = 0 \tag{4}$$

And then

$$N_2 = \frac{A_H}{W} \tag{5}$$

Equation (5) is employment in the second period when productivity turns out to be high: in this case employment does not depends on firing costs *z*.

If we have A_L (with probability *p*), the firms necessarily want to decrease N_1 and $N_1 - N_2 > 0$. Then second period profits are:

$$\pi_2 = A_L \log N_2 - WN_2 - z \cdot \max(N_1 - N_2, 0) = A_L \log N_2 - WN_2 - z \cdot (N_1 - N_2) \tag{6}$$

In this case optimal employment entails

$$\frac{\partial \pi_2}{\partial N_2} = \frac{A_L}{N_2} - W + z = 0 \tag{7}$$

And then

$$N_2 = \frac{A_L}{W-z} \qquad (8)$$

Equation (8) is employment in the second period when productivity turns out to be low: in this case employment depends positively on firing costs *z*.

First period decisions

Given (5) and (8) we have that the total expected profits are:

$$E(\pi) = \log N_1 - WN_1 + pA_L \log \frac{A_L}{W-z} + (1-p)A_H \log \frac{A_H}{W} - p \cdot z \cdot WN_2 - p \cdot z \cdot \left(N_1 - \frac{A_L}{W-z}\right) \qquad (9)$$

In the first period, firms choose N_1 optimally to maximise the above expected profits. Then

$$\frac{\partial E(\pi)}{\partial N_1} = \frac{1}{N_1} - W - p \cdot z = 0 \qquad (10)$$

And then

$$N_1 = \frac{1}{W + p \cdot z} \qquad (11)$$

Equation (11) is employment in the first period and it depends negatively on firing costs *z*.