



## Statistical Methods for the Evaluation of Labour Policies.

Co-funded by the  
Erasmus+ Programme  
of the European Union



Dott.ssa Irene Brunetti<sup>1</sup>

<sup>1</sup>[i.brunetti@inapp.org](mailto:i.brunetti@inapp.org)

*National Institute for Public Policies Analysis*

(INAPP - Roma)

29/10/2018

# Introduction

---

## Example 1

Suppose we have a group of unemployed individuals who are in training and who are looking for a job. Six months after the end of the training program we check their employment situation and find that 40% of the group is working.

- Can we conclude that 40% of those that were unemployed before the training found work *because* of the training program?
- In order to make a comprehensive evaluation one needs to take into account many things  $\Rightarrow$  how can we isolate the effect so it can be attributed solely on the treatment (the training)?

# Why evaluate?

---

- Programs and policies are usually designed to change outcomes, for example, to increase incomes, to reduce unemployment, or to raise human capital.
- Understand if these changes are achieved is a crucial question.
- Evaluate “programs/policies” helps:
  - to improve efficiency of the policy;
  - to optimize the use of resources;
  - to favour transparency;
  - to reduce the complexity of some processes.

# What is causality?

A causal question is a **simple question involving the relationship between two theoretical concepts**: a cause and an effect.

- Cause  $\implies$  Effect?
- Does  $X$  cause  $Y$ ?
- If  $X$  causes  $Y$ , how large is this effect?

# Why is causality so important?

---

- The primary aim of all sciences.
- Understanding of causal relationships leads to accurate predictions of the future.
- It provides the scientific basis for policy intervention.
- It advances our theoretical knowledge of the world.

# Program evaluation

---

Program evaluation is the systematic process of studying a program, or a policy, to discover how well it is working to achieve intended goals.

The **goal** in program evaluation is to **assess the causal effect of public policy interventions**.

Examples include effects of:

- Job training programs on earnings and employment.
- Class size on test scores.
- Minimum wage on employment.
- Military service on earnings and employment.

## Preliminary questions

---

To measure the effect of a program/policy is essentially to understand:

- **Effect about what?:** identify the **outcome-variable**.
- **Effect of what?:** specify the **treatment-variable**.
- **Effect for whom?:** identify the **target population**.

## Rubin Causal Model, (Rubin and Imbens, 2010)

---

The Rubin Causal Model (RCM) is a formal mathematical framework for causal inference.

Two essential parts to the RCM:

- the use of "*potential outcomes*" to define causal effects in all situations.
- An explicit probabilistic model for the "*assignment of treatments*" to units  $\Rightarrow$  the *assignment mechanism*.



# Causality with potential outcome

## Treatment:

$D_i$ : Indicator of treatment for unit  $i$

$$D_i = \begin{cases} 1, & \text{if unit } i \text{ received the treatment} \\ 0, & \text{otherwise} \end{cases}$$

## Outcome:

### Potential Outcomes

- $Y_{0i}$ : Potential outcome for unit  $i$  without treatment (the **counterfactual**);
- $Y_{1i}$ : Potential outcome for unit  $i$  with treatment.

### Observed Outcome

- $Y_i$

# Causality with potential outcome

## Observed Outcomes

Observed outcomes are realized as:

$$Y_i = Y_{1i}D_i + Y_{0i}(1 - D_i) \quad (1)$$

$$\text{or } Y_i = \begin{cases} Y_{1i}, & \text{if } D_i = 1 \\ Y_{0i}, & \text{if } D_i = 0 \end{cases}$$

## Treatment Effect

The treatment effect (or causal effect) on the outcome for unit  $i$  is the difference between his/her two potential outcomes:

$$\Delta_i = Y_{1i} - Y_{0i}$$

# An example

Imagine a population with 4 units

Table: A numerical example

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$	$\Delta_i$
1	1	3	3	0	3
2	1	1	1	1	0
3	0	0	1	0	1
4	0	1	1	1	0

# Identification problem for causal inference

## Fundamental Problem of Causal Inference

Cannot observe both potential outcomes  $(Y_{1i}, Y_{0i}) \implies$  How can we find  $Y_{1i} - Y_{0i}$ ?

A large amount of homogeneity would solve this problem:

- $(Y_{1i}, Y_{0i})$  constant across individuals;
- $(Y_{1i}, Y_{0i})$  constant across time.
- Unfortunately, often there is a large degree of heterogeneity in the individual responses to participation in public programs/policy.

## Connection to linear model

### How to estimate the effect of treatment?

- Suppose we wish to measure the impact of treatment on an outcome,  $Y$ . For the moment, we abstract from other covariates that may impact on  $Y$ .
- $D$  is the treatment indicator: a dummy variable assuming the value 1 if the individual has been treated and 0 otherwise.
- The potential outcomes for individual  $i$  at any time  $t$  are denoted by  $Y_{1it}$  and  $Y_{0it}$ .
- These outcomes are specified as:

$$\begin{aligned} Y_{1it} &= \beta + \rho_i + \epsilon_{it} \text{ if } D_{it} = 1 \\ Y_{0it} &= \beta + \epsilon_{it} \text{ if } D_{it} = 0 \end{aligned} \tag{2}$$

where  $\beta$  is the intercept parameter,  $\rho_i$  is the effect of treatment on individual  $i$  and  $\epsilon$  is the unobservable component of  $Y$ .

## Connection to linear model

The observable outcome is then:

$$Y_i = Y_{1i}D_i + Y_{0i}(1 - D_i)$$

so that

$$Y_{it} = \beta + \rho_i D_{it} + \epsilon_{it} \quad (3)$$

where  $\mathbf{E}(\epsilon) = 0$  and  $\text{cov}(\epsilon, D) = 0$ . Estimating Eq.(3) by OLS we obtain the estimate of the causal effect of D.

## Quantities of interest

Instead of the individual treatment effect, we might be interested in the **average treatment effect (ATE)**:

$$\begin{aligned}\alpha_{ATE} &= \mathbf{E}[Y_{1i} - Y_{0i}] \\ &= \mathbf{E}[Y_{1i}] - \mathbf{E}[Y_{0i}]\end{aligned}\tag{4}$$

- If  $ATE > 0 \implies \mathbf{E}[Y_{1i}] > \mathbf{E}[Y_{0i}] \implies$  The policy is good;
- if  $ATE < 0 \implies \mathbf{E}[Y_{1i}] < \mathbf{E}[Y_{0i}] \implies$  The policy is bad;
- if  $ATE = 0 \implies \mathbf{E}[Y_{1i}] = \mathbf{E}[Y_{0i}] \implies$  The policy has no impact

**BUT** we can not find the ATE because of the unobserved potential outcomes.

# Average Treatment Effect (ATE)

Imagine a population with 4 units:

Table: A numerical example

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$	$\Delta_i$
1	1	3	3	0	3
2	1	1	1	1	0
3	0	0	1	0	1
4	0	1	1	1	0
$E[Y_1]$			1.5		
$E[Y_0]$			0.5		
$E[\Delta]$			1		

$$\alpha_{ATE} = E[\Delta] = 3 * (1/4) + 0 * (1/4) + 1 * (1/4) + 0 * (1/4) = 1$$



## Quantities of interest

We might also be interested in the **average treatment effect on the treated** (ATET):

$$\begin{aligned}\alpha_{ATET} &= \mathbf{E}[Y_1 - Y_0 | D = 1] \\ &= \mathbf{E}[Y_{1i} | D = 1] - \mathbf{E}[Y_{0i} | D = 1]\end{aligned}\tag{5}$$

**BUT** we can not find the ATET because of unobserved potential outcomes.

# Average Treatment Effect on the Treated (ATET)

Imagine a population with 4 units:

Table: A numerical example

$i$	$D_i$	$Y_i$	$Y_{1i}$	$Y_{0i}$	$\Delta_i$
1	1	3	3	0	3
2	1	1	1	1	0
3	0	0	1	0	1
4	0	1	1	1	0
$E[Y_1 D=1]$			2		
$E[Y_0 D=1]$			0.5		
$E[\Delta 1]$					1.5

$$\alpha_{ATET} = E[\Delta|D=1] = 3 * (1/2) + 0 * (1/2) = 1.5$$

# ATE vs ATET

---

- ATE is relevant when the treatment has universal applicability.
- ATET is relevant when we want to consider the average gain treatment for the treated.

## Estimating ATE

Both  $Y_{0i}$  and  $(Y_{0i}|D = 1)$  are unobserved  $\implies$  we can estimate the ATE as:

$$\begin{aligned}\hat{ATE} &= E[Y_{1t} - Y_{0c}] \\ &= E[Y_{1t}] - E[Y_{0c}]\end{aligned}\tag{6}$$

where  $Y_{1t}$  is the outcome of treated and  $Y_{0c}$  is the outcome of untreated (the **control group**). Both **quantities are observed**.

We basically find the average  $Y$  for observations that received treatment and average  $Y$  for observations that received control.

## Selection bias

What are we measuring if we compare the outcomes for the treated to the untreated? Is this the causal effect (i.e., the effect of treatment on the outcome)? NO

$$\begin{aligned}
 \mathbf{E}[Y|D = 1] - \mathbf{E}[Y|D = 0] &= \mathbf{E}[Y_1|D = 1] - \mathbf{E}[Y_0|D = 0] \\
 &= \mathbf{E}[Y_1|D = 1] - \mathbf{E}[Y_0|D = 1] + \mathbf{E}[Y_0|D = 1] - \mathbf{E}[Y_0|D = 0] \\
 &= \underbrace{\mathbf{E}[Y_1 - Y_0|D = 1]}_{\text{ATET}} + \underbrace{\mathbf{E}[Y_0|D = 1] - \mathbf{E}[Y_0|D = 0]}_{\text{BIAS}}
 \end{aligned} \tag{7}$$

where the bias term is the difference between no-treatment outcomes of individuals that are treated and those that are not treated.

- If no selection bias, then we get the ATET. The ATET is of interest but note that it is not the same as the ATE (except in special cases).
- If there is selection bias, estimate of the ATE based on comparing the average outcomes of the treated to the untreated will be misleading (biased).

# Selection Bias

---

The **bias term is not likely to be zero** for most public policy.

The **goal** is to minimize/eliminate it.

## Sources of bias

- **Self-selection**  $\Rightarrow$  positive bias  $\Rightarrow$  causal effect will be overstated;
- **Targeting**  $\Rightarrow$  negative bias  $\Rightarrow$  causal effect will be understated;
- **Observables vs. Unobservables.**

# Assumptions for unbiased estimate

---

What assumptions do we need for the estimate to be unbiased?

- Stable Unit Treatment Value Assumption (SUTVA);
- Unconfoundedness/ignorability.;

# Stable Unit Treatment Value Assumption (SUTVA)

The **stable unit treatment value assumption** (SUTVA) assumes that:

- the treatment status of any unit does not affect the potential outcomes of the other units (**non-interference**);
- the treatments for all units are comparable (**no variation in treatment**).

## Violations:

- Vaccination (interference);
- Fertilizer A and B on crop yield, but each fertilizer has a lot of versions (variation in treatment).

This assumption may be problematic, so we should choose the units of analysis to minimize interference across units.



## Unconfoundedness/Ignorability

### Conditional independence assumption:

Given a vector of observable variables  $\mathbf{x}$  (vector of covariates), the assumption states that, conditional on  $\mathbf{x}$ , the outcomes are independent of treatment:

$$(Y_1, Y_0) \perp D | \mathbf{x} \quad (8)$$

### Unconfoundedness (strong ignorability):

$$(Y_1, Y_0) \perp D \quad (9)$$

Treatment assignment is **independent** of the outcomes ( $Y$ ). Technically, unconfoundedness is a stronger assumption. Most people just say ignorability.

### Violations:

- Omitted Variable Bias

# Assignment mechanism

---

## Assignment Mechanism

Assignment mechanism is the procedure that determines which units are selected for treatment intake. Examples include:

- random assignment;
- selection on observables: **matching, regression discontinuity;**
- selection on unobservables: **Diff-in-Diff, control function approach, instrumental variable estimation.**

Most models of causal inference attain identification of treatment effects by restricting the assignment mechanism in some way.

## Key ideas

---

- Causality is defined by potential outcomes, not by realized (observed) outcomes;
- Estimation of causal effects of a treatment (usually) starts with studying the assignment mechanism.

Statistical Methods for the Evaluation of Labour Policies

*Irene Brunetti – i.brunetti@inapp.org*

---



INAPP - Istituto Nazionale per l'Analisi delle Politiche Pubbliche  
Corso d'Italia, 33 - 00198 Roma - tel. +39.06.85447.1 - [www.inapp.org](http://www.inapp.org)